Co-Annihilation of Heavy Particles in Thermal Environment

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based on
SK, M. Laine (ITP, U of Bern),
JHEP1607(2016)143(arxiv:1602.08105) and
arXiv:1609.00474

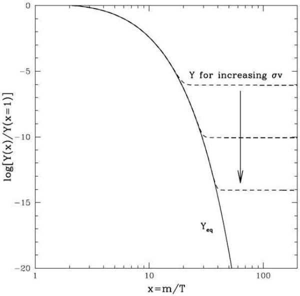
Outline

1. Introduction
2. Formalism
3. Conclusion

Decoupling in heavy-ion collisions/cosmology

• chemical equilibriation of heavy quarks in Quark-Gluon Plasma (QGP) ?

• co-annihilation of dark matter in early universe: dark matter decouples when pair annihilation rate is not fast enough to keep up with thermal equilibrium distribution



$$n_{\rm eq} \sim \int_{\mathbf{p}} e^{-\beta E_{\mathbf{p}}} = \left(\frac{MT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{M}{T}} \qquad (1)$$

Decoupling in heavy-ion collisions/cosmology

• and rough estimate of decoupling temperature is for dark matter is, Hubble rate $\sim$ co-annihilation rate

$$H \sim n < \sigma v > \;\rightarrow\; \frac{T^2}{M_{\rm pl}} \sim (\frac{MT}{2\pi})^{\frac{3}{2}} e^{-\frac{M}{T}} \frac{\alpha^2}{M^2} \tag{1}$$

for $\alpha \sim 0.01$, $T \sim \frac{M}{25}$

Decoupling in heavy-ion collisions/cosmology

• “Sommerfeld effect” enhances co-annihilation (heavy quark co-annihilation in QGP and dark matter (WIMP/SIMP) co-annihilation in cosmology) (e.g, Hisano et al, hep-ph/0612049)

• thermal effect (producing mass shift, thermal width, mixing angle modification) can be $\mathcal{O}(1)$ effect

• bound states can be disturbed by this $\mathcal{O}(1)$ effect

• such effects can be studied through the change of spectral function/or thermal correlator

Decoupling in heavy-ion collisions/cosmology

• for example, modification of heavy quark potential in thermal environment (cf. M. Laine et al, hep-ph/0611300).

$$V(r) = -\alpha_s \left[m_D + \frac{\exp(-m_D r)}{r} \right] \tag{1}$$

and

$$\Gamma(r) = 2\alpha_s T \int_0^\infty dx \, \frac{x}{(1+x^2)^2} \left[1 - \frac{\sin(x m_D r)}{x m_D r} \right] \tag{2}$$

Formalism – requirement

- for QCD, non-perturbative definition for the chemical/kinetic equilibriation rate is necessary

- equilibriation rate is a real-time quantity

- lattice gauge theory is a method which can calculate non-perturbative quantities using first principles of quantum field theory

- lattice gauge theory is defined on a Euclidean space and has difficulty in calculating real-time quantity

Formalism – requirement

• the number density (n) of heavy quarks or dark matter (Boltzmann equation)

$$(\partial_t + 3H)n \simeq -c(n^2 - n_{eq}^2) \tag{3}$$

in linearized form

$$(\partial_t + 3H)n = -\Gamma_{\text{chem}}(n - n_{eq}) + \mathcal{O}(n - n_{eq})^2 \tag{4}$$

where $\Gamma_{\text{chem}} = 2cn_{eq}$, chemical equilibriation rate

$\Gamma_{\rm chem}$ as a transport coefficient

• chemical equilibriation as a transport coefficient (D. Bödeker, M. Laine, JHEP07 (2012) 130, 01 (2013) 037)

• treat the approach to the equilibrium as a Langevin process

$$\delta\dot{n}(t) = -\Gamma_{\rm chem}\delta n(t) + \xi(t) \tag{5}$$

$$\langle\langle \xi(t)\xi(t')\rangle\rangle = \Omega_{\rm chem}\delta(t-t'), \qquad \langle\langle \xi(t)\rangle\rangle = 0 \tag{6}$$

where $\delta n(t)$ is the deviation from the equilibrium and $\xi(t)$ is a stochastic noise

$$\delta n(t) = \delta n(t_0)e^{-\Gamma_{\rm chem}(t-t_0)} + \int_{t_0}^{t} dt' e^{\Gamma_{\rm chem}(t'-t)}\xi(t') \tag{7}$$

$\Gamma_{\rm chem}$ as a transport coefficient

- for heavy quarks in QCD, quarkonium decay can be expressed in terms of long distance matrix element times short distance partonic cross section (cf. E. Braate et al, hep-ph/9407339)

- in thermal environment, through linear response theory

$$n_{eq}\Gamma_{\rm chem} = \frac{8\alpha_s^2}{M^2}\frac{1}{\mathcal{Z}}\sum_m e^{-E_m/T}\langle m|\psi^\dagger\chi\chi^\dagger\psi|m\rangle \qquad (5)$$

$\Gamma_{\rm chem}$ as a transport coefficient

- thermal average can be expressed in terms of a Wightman function

$$
\begin{aligned}
\gamma &= \frac{1}{\mathcal{Z}} \sum_m e^{-E_m/T} \langle m | \psi^\dagger \chi \chi^\dagger \psi | m \rangle \\
&= \langle \psi^\dagger \chi(0, \mathbf{0}) \chi^\dagger \psi(0, \mathbf{0}) \rangle \\
&= \int_{\omega, \mathbf{k}} \int_{t, \mathbf{x}} e^{i(\omega t - \mathbf{k} \cdot \mathbf{x})} \langle \psi^\dagger \chi(0, \mathbf{0}) \chi^\dagger \psi(t, \mathbf{x}) \rangle
\end{aligned} \tag{5}
$$

Γ_{chem} as a transport coefficient

$$\Pi_{<}(\omega,\mathbf{k}) = 2n_B(\omega)\rho(\omega,\mathbf{k}) \sim 2e^{-\omega T}\rho(\omega,\mathbf{k}) \tag{5}$$

for non-relativistic case

$$\omega = 2M + \frac{\mathbf{k}^2}{4M} + E' \tag{6}$$

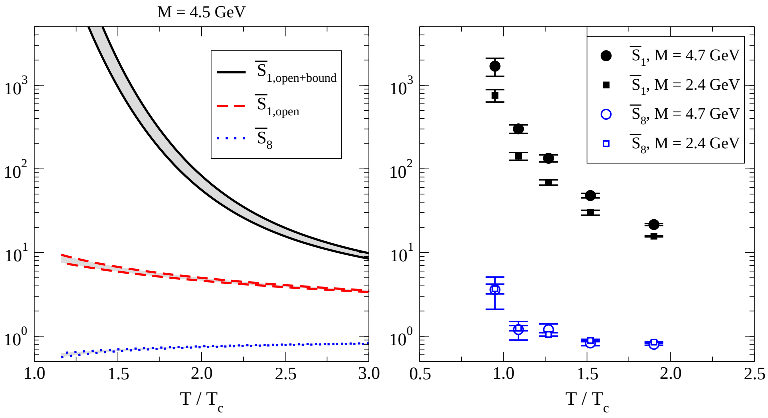
and

$$\gamma \sim (\frac{MT}{2\pi})^{\frac{3}{2}} e^{-\frac{M}{T}} \frac{\alpha^2}{M^2} \int_{-\Lambda}^{\infty} \frac{dE'}{\pi} e^{-E'/T} \rho(E') \tag{7}$$

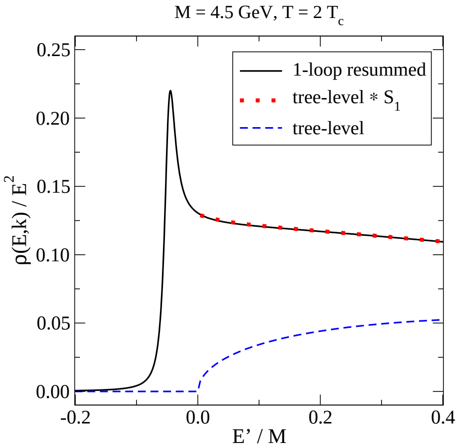
$\Gamma_{\rm chem}$ as a transport coefficient

- compute thermal (full or HTL) gauge field self-energy
- determine corresponding time-order propagator
- Fourier transform of the potential and the width
- solve for the spectral function, $\rho(E') = ImG(E'; \mathbf{0}, 0)$
- laplace transform with weight $e^{-E'/T}$ for $\langle m|O|m\rangle$ (e.g., $O = \psi^\dagger\chi\chi^\dagger\psi$)

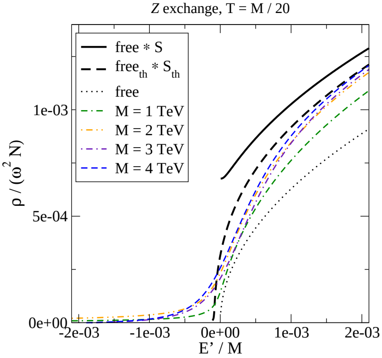
QCD case


M = 4.5 GeV
$\overline{S}_{1,open+bound}$
$\overline{S}_{1,open}$
$\overline{S}_8$
10^3
10^2
10^1
10^0
1.0
1.5
2.0
2.5
3.0
T / T$_c$
$\overline{S}_1$, M = 4.7 GeV
$\overline{S}_1$, M = 2.4 GeV
$\overline{S}_8$, M = 4.7 GeV
$\overline{S}_8$, M = 2.4 GeV
0.5
1.0
1.5
2.0
2.5
T / T$_c$

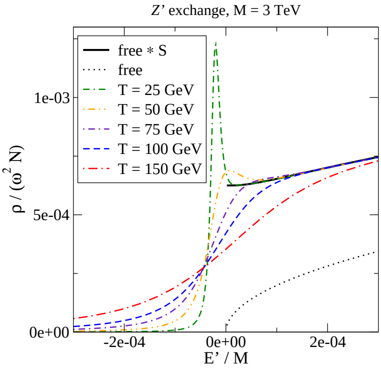
QCD case


M = 4.5 GeV, T = 2 T_c
1-loop resummed
tree-level * S_1
tree-level
ρ(E,k) / E^2
0.25
0.20
0.15
0.10
0.05
0.00
-0.2
0.0
0.2
0.4
E' / M

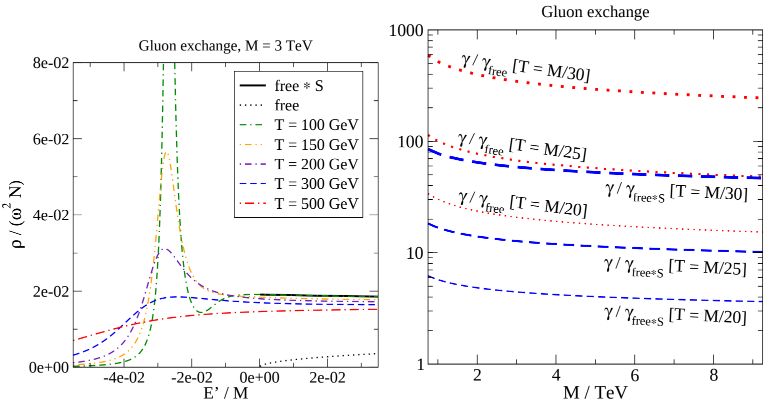
Z boson exchange: no bound state case



Z' boson exchange: bound states melt below freeze-out



Gluon exchange between gluinos: QCD-like



Conclusion

- a real time quantity, chemical equilibriation rate, is calculated non-perturbatively using Euclidean lattice without analytic continuation

- thermal Sommerfeld effect for bottomonium co-annihilation is calculated using lattice NRQCD and is found to be two orders of magnitude larger than perturbative estimate

- for weak interaction (Z boson exchange), existing conclusion on the Sommerfeld effect is confirmed

- similarly, in “strongly interacting” dark matter scenario, the bound state effect enhances co-annihilation of dark matter far beyond a naive perturbative estimate.